Utilising the topology of Large Scale Structure to constrain cosmology

Stephen Appleby – KIAS Research Fellow
Collaborators – Changbom Park, Sungwook Hong, Juhan Kim

7th KIAS Workshop on Cosmology
and Structure Formation
30th Oct - 4th Nov, 2016

S. Appleby et al., ApJ
Submitted (2016)

Can we use Minkowski functionals to constrain cosmology?

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Overview

1) Introduction to Minkowski Functionals
   - Definition
   - Gaussian fields in two and three dimensions

2) Numerical reconstruction of Minkowski Functionals
   - Two dimensional field
   - Three dimensional field

3) What can we measure with Minkowski Functionals?
   - Amplitude, shape of genus curve

4) Systematics
   - RSD
   - Pixel effects
   - Shot noise/Halo bias

5) Future directions

Minkowski Functionals

- The Minkowski Functionals (MF's) are a set of scalar quantities that describe the morphology and topology of an excursion set.
- In N dimensions, there exist N+1 MF's. They are related to simple quantities such as the volume and surface area of the excursion set.
- In two dimensions, the Minkowski Functionals correspond to the surface area, perimeter and genus.

d	1	2	3
V_0	length	area	volume
V_1	χ	circumference	surface area
V_2	–	χ	total mean curvature
V_3	–	–	χ

Minkowski Functionals - 2D

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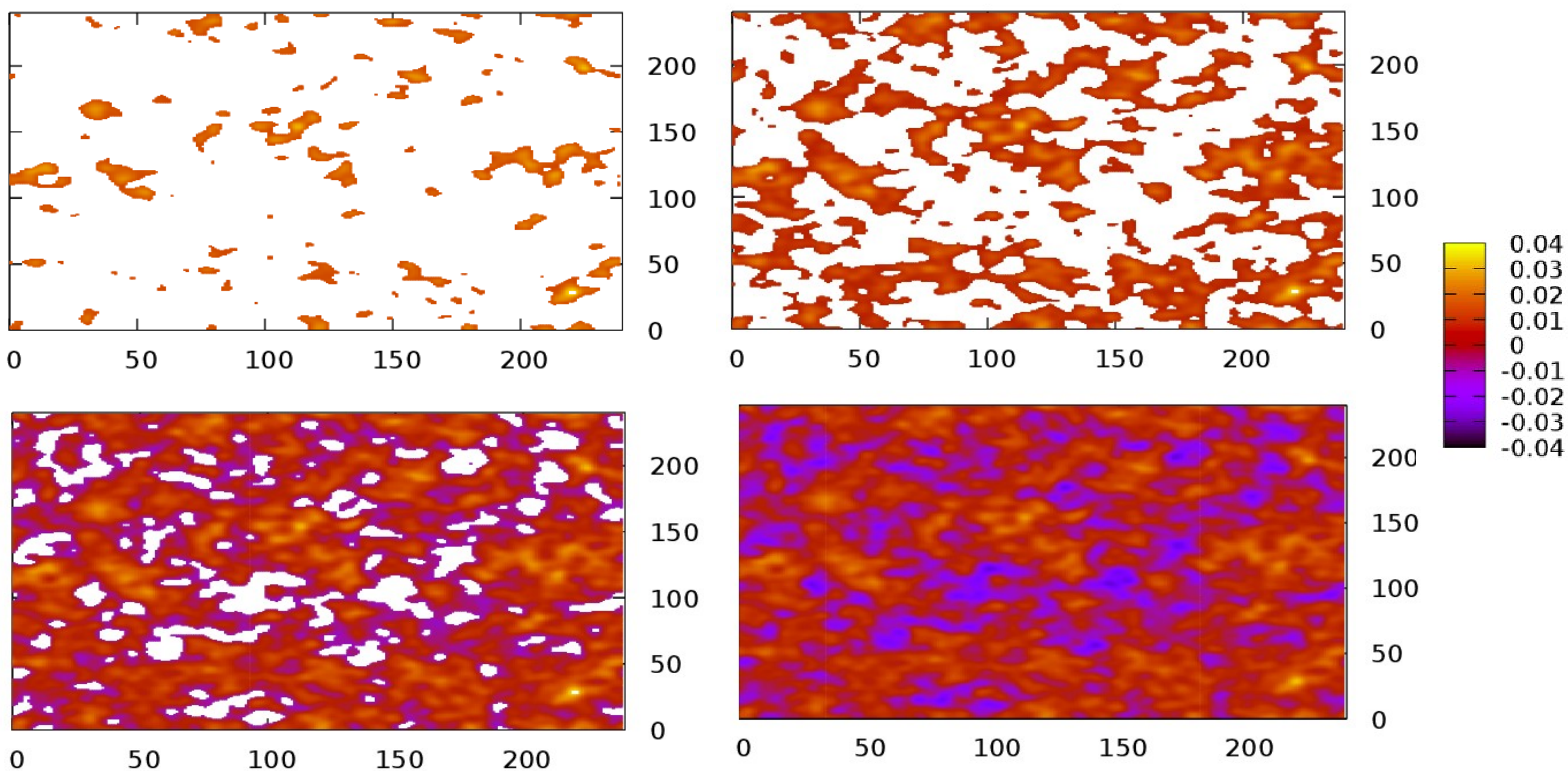
Apply a density cut

V_0 = Area

V_1 = Perimeter

V_2 = Genus = number of hot spots – number of cold spots

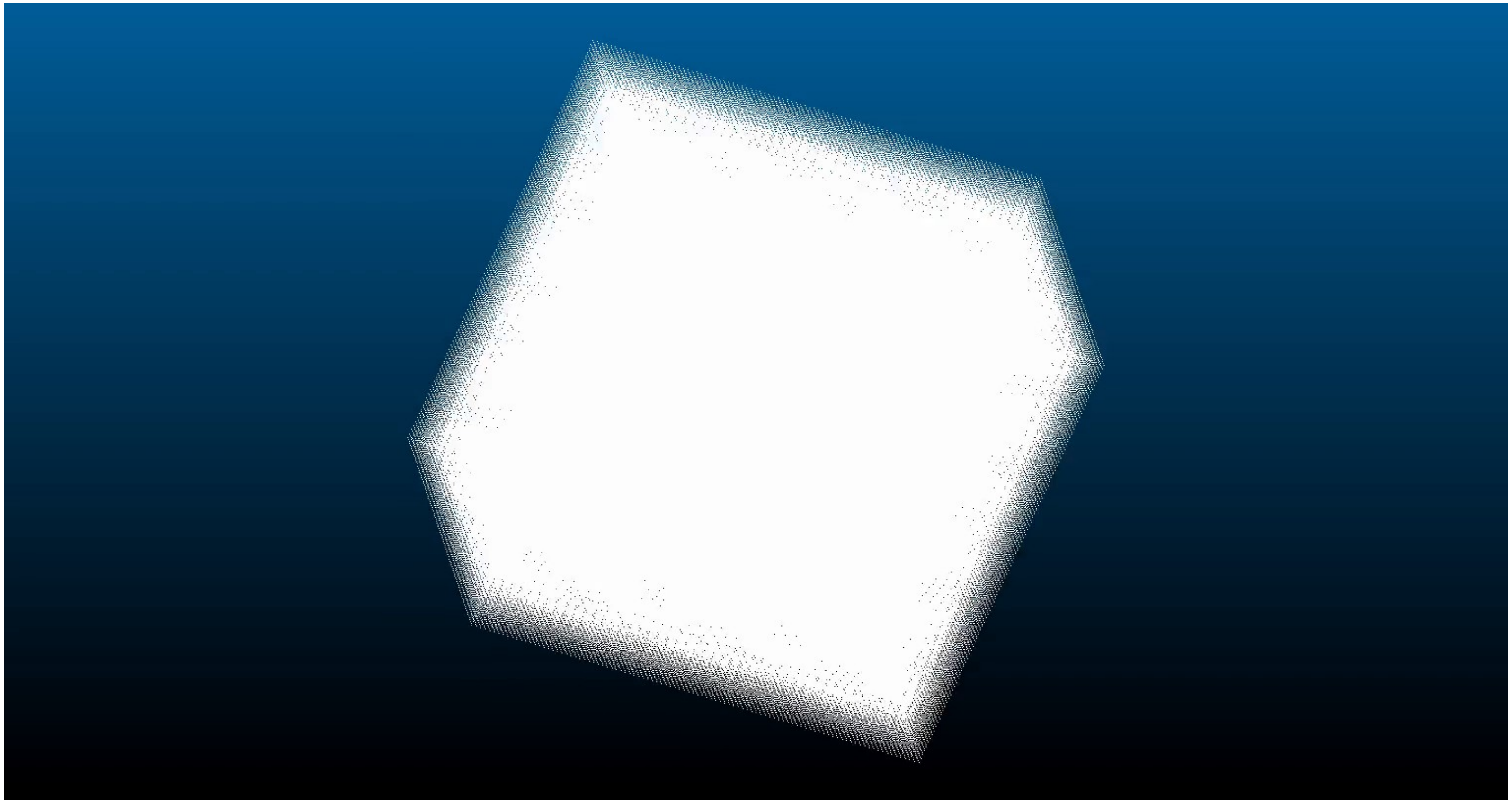


Minkowski Functionals - 3D

Three dimensions -

V_0 = Volume V_1 = Area

V_2 = Mean Curvature V_3 = Genus



Minkowski Functionals - 3D

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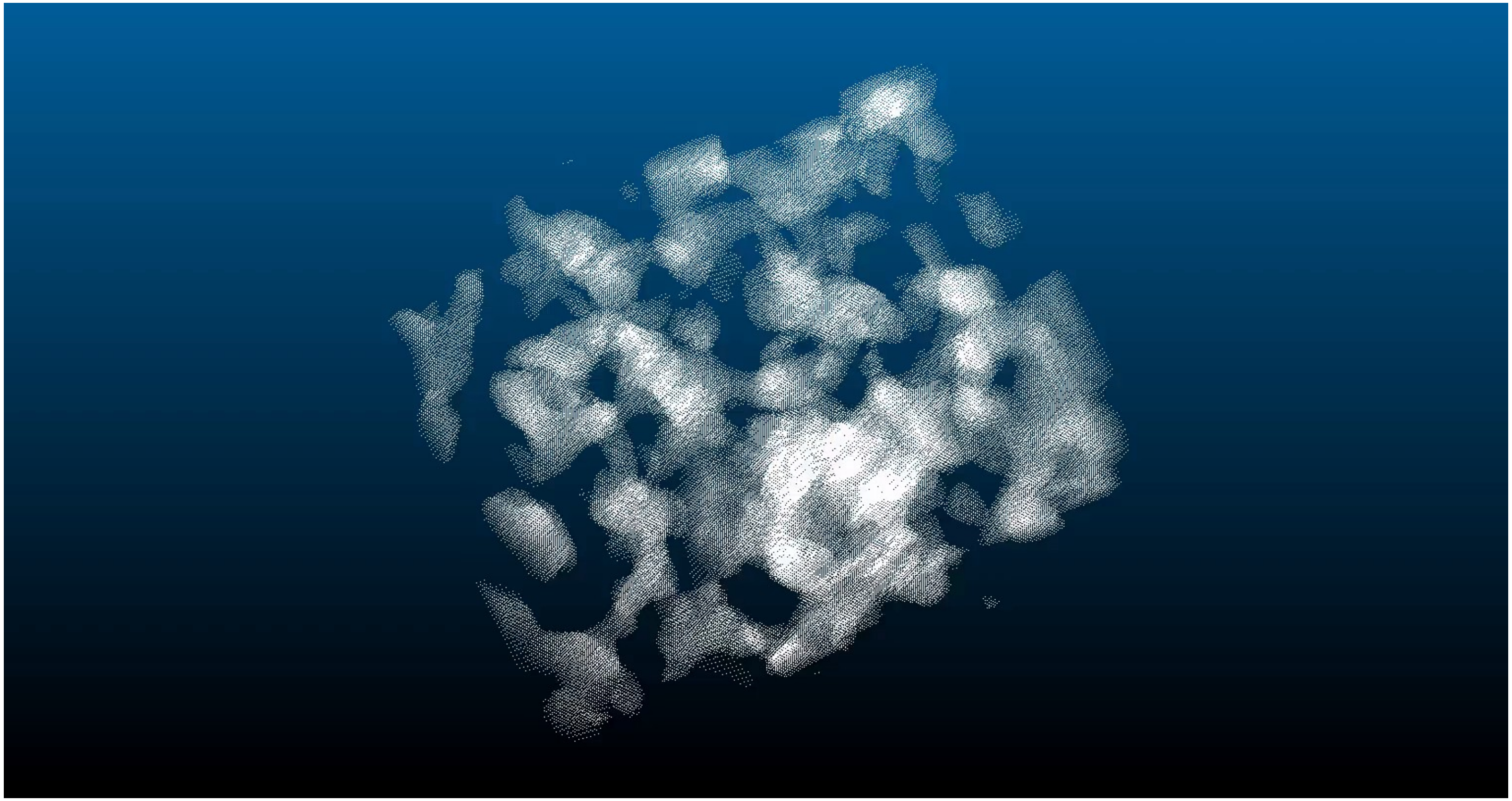

Minkowski Functionals - 3D

Three dimensions -

V_0 = Volume V_1 = Area

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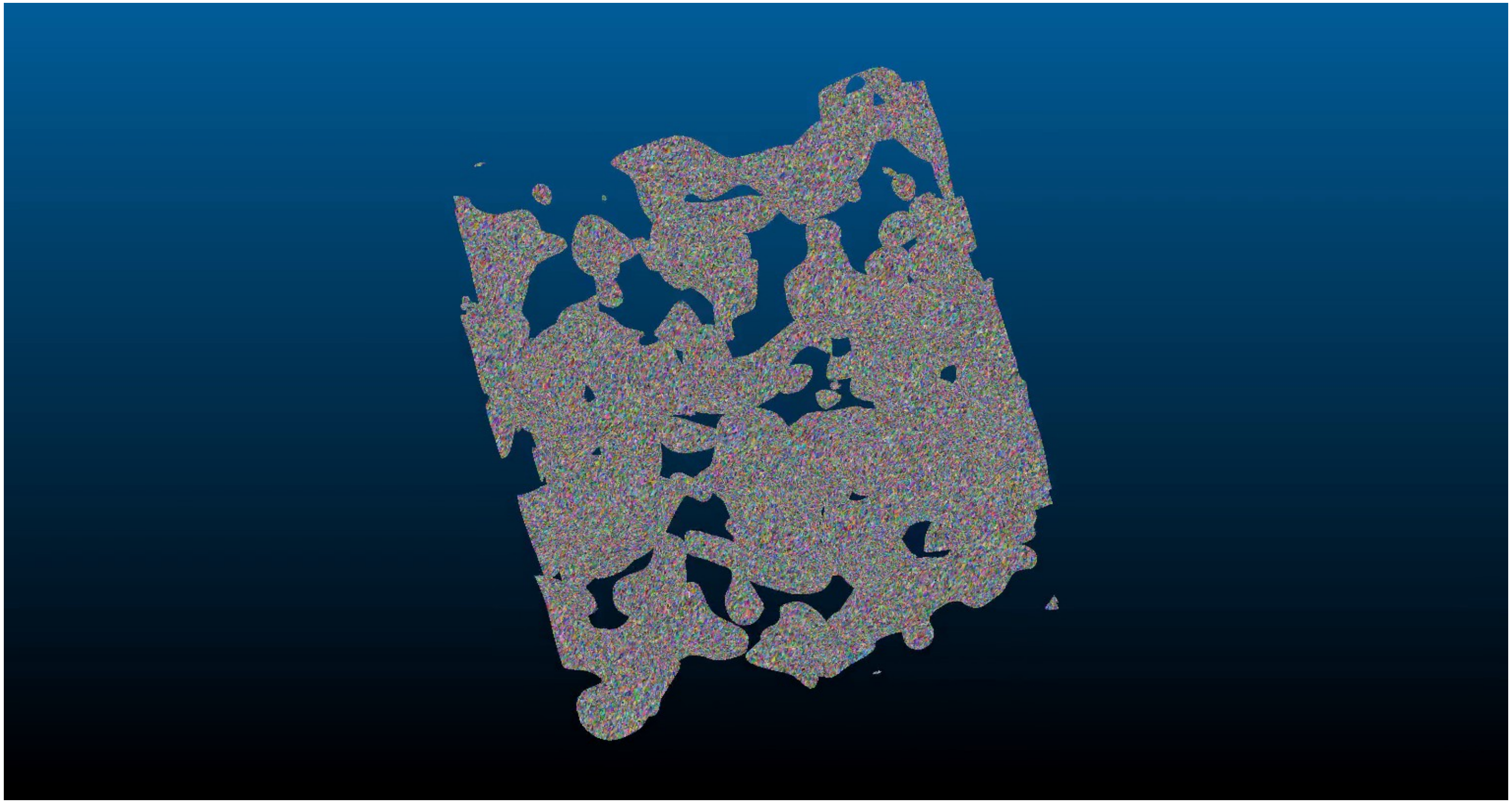
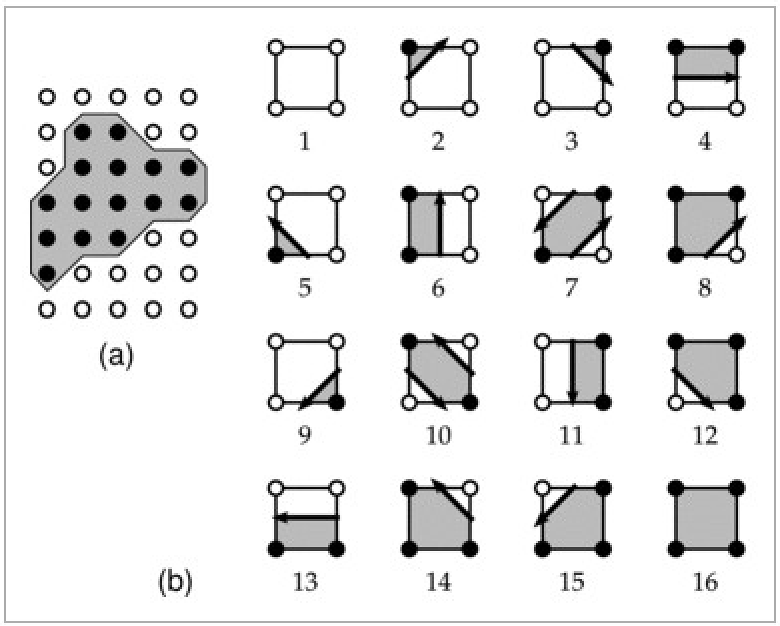
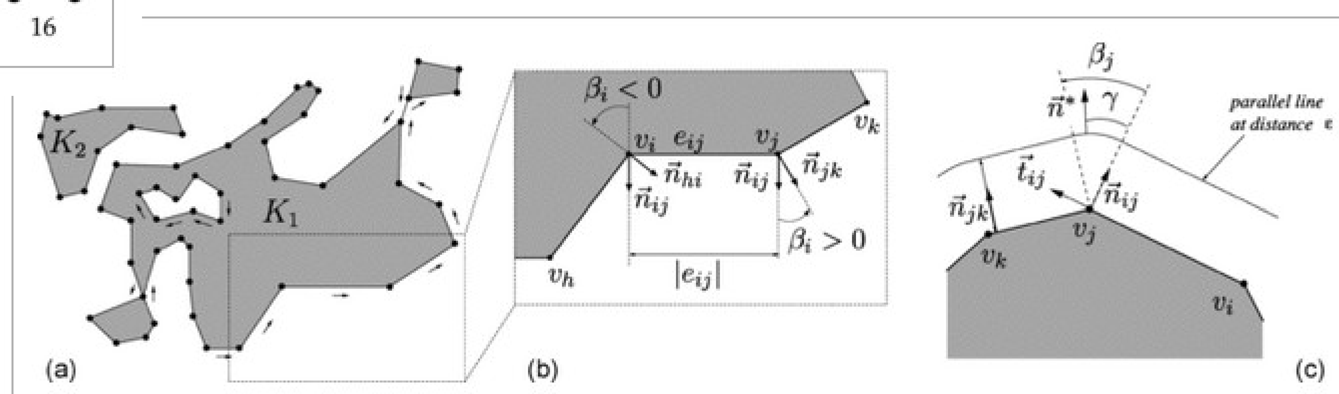


Minkowski Functionals 2D - Numerical Implementation

- Marching Squares Algorithm (Taken from Turk et al 2009)



Name	Definition	Formula (or linear dependence)
W_0	$\int \mathrm{d}^2 r$	$= \sum_{(i,j)} \frac{1}{4} \lvert e_{ij} \rvert \cdot \left(\vec{n}_{ij} \cdot (\vec{v}_i + \vec{v}_j) \right)$
W_1	$\int \frac{1}{2}\, \mathrm{d}r$	$= \sum_{(i,j)} \frac{1}{2} \lvert e_{ij} \rvert$
W_2	$\int \frac{1}{2} \kappa\, \mathrm{d}r$	$= \sum_i \frac{1}{2} \beta_i$



Minkowski Functionals 3D - Numerical Implementation

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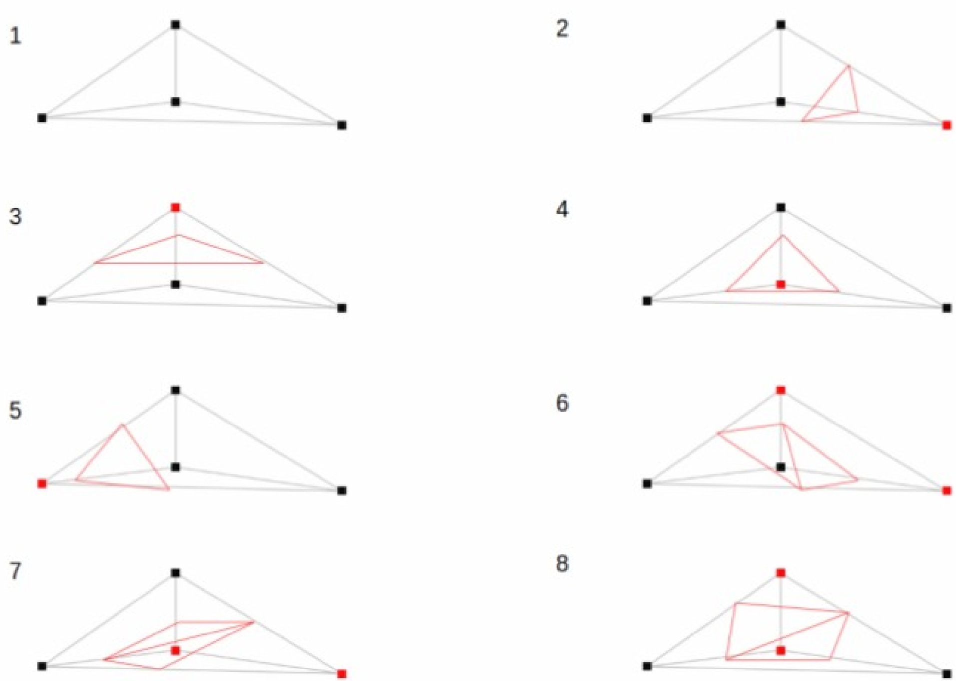
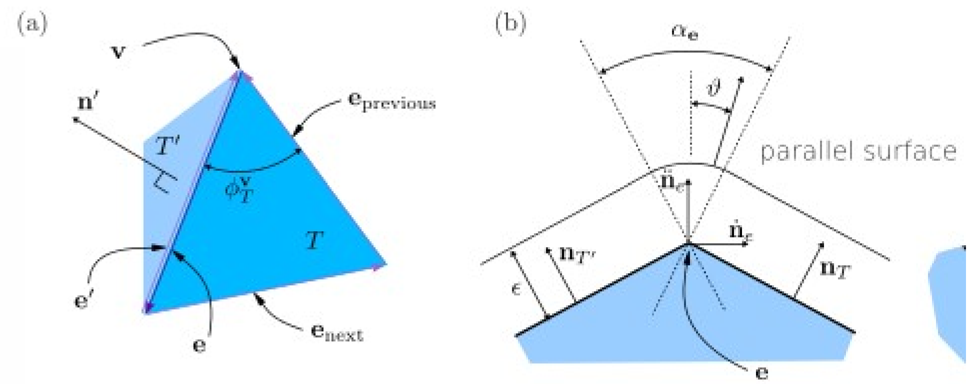
- Marching Tetrahedra (Taken from Turk et al 2010)



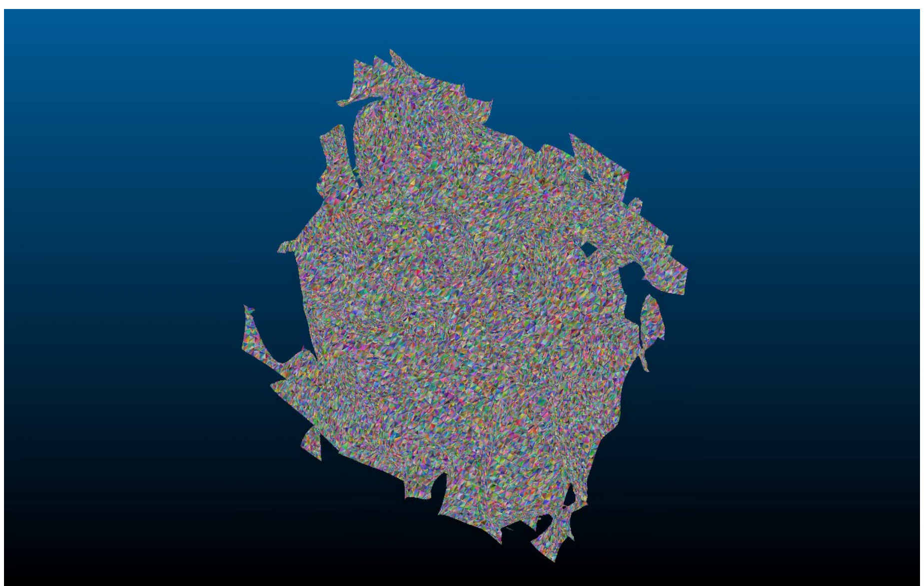


W_0	$\int_K \mathrm{d}V$	$\frac{1}{3}\sum_{T\in\mathcal{F}_2}\langle \mathbf{C}_T, \mathbf{n}_T\rangle \lvert T\rvert$
W_1	$\frac{1}{3}\int_{\partial K}\mathrm{d}A$	$\frac{1}{3}\sum_{T\in\mathcal{F}_2}\lvert T\rvert$
W_2	$\frac{1}{3}\int_{\partial K} G_2\,\mathrm{d}A$	$\frac{1}{12}\sum_{\mathbf{e}\in\mathcal{F}_1}\lvert\mathbf{e}\rvert\,\alpha_{\mathbf{e}}$
W_3	$\frac{1}{3}\int_{\partial K} G_3\,\mathrm{d}A$	$\frac{1}{3}\sum_{\mathbf{v}\in\mathcal{F}_0}(2\pi - \sum_{T\in\mathcal{F}_2(\mathbf{v})}\phi_T^{\mathbf{v}})$

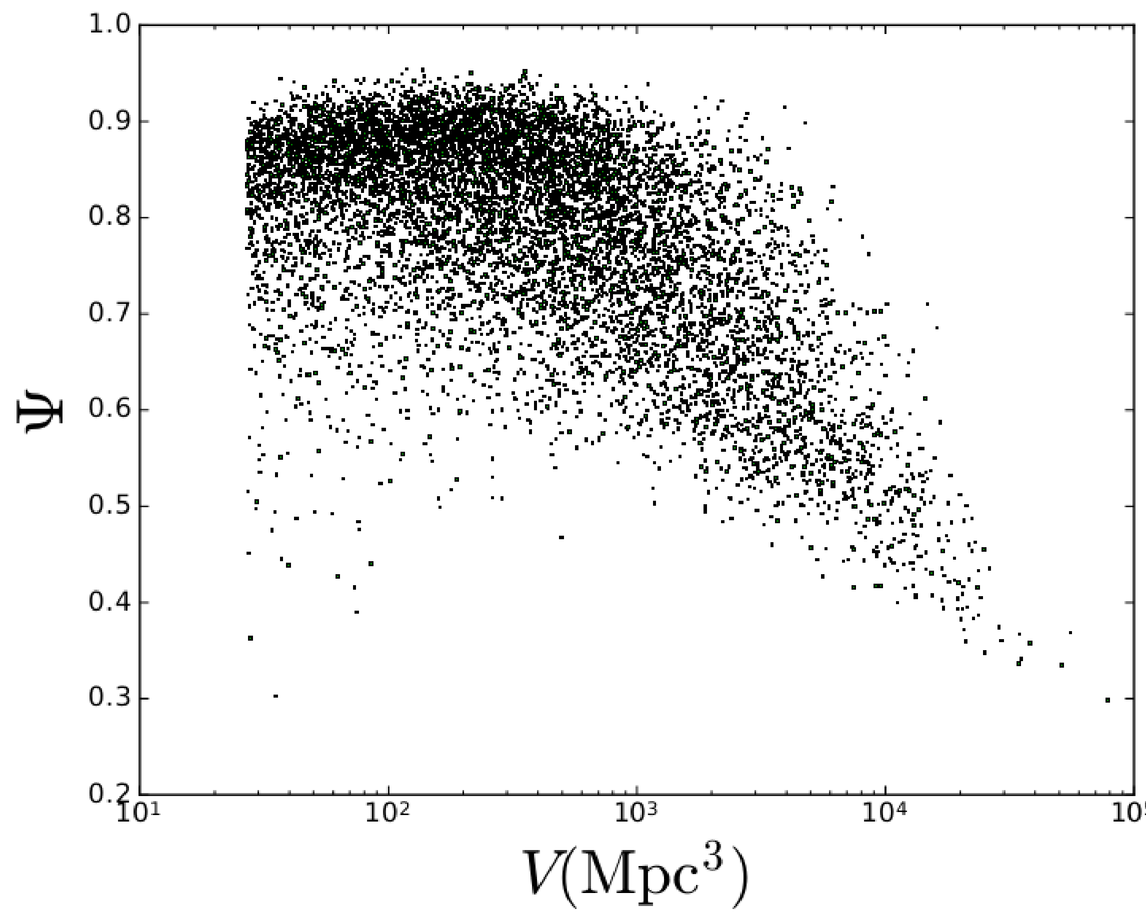
Other Uses

Can calculate the morphological properties of any subset of a density field

Example – Filaments (Gaussian field)

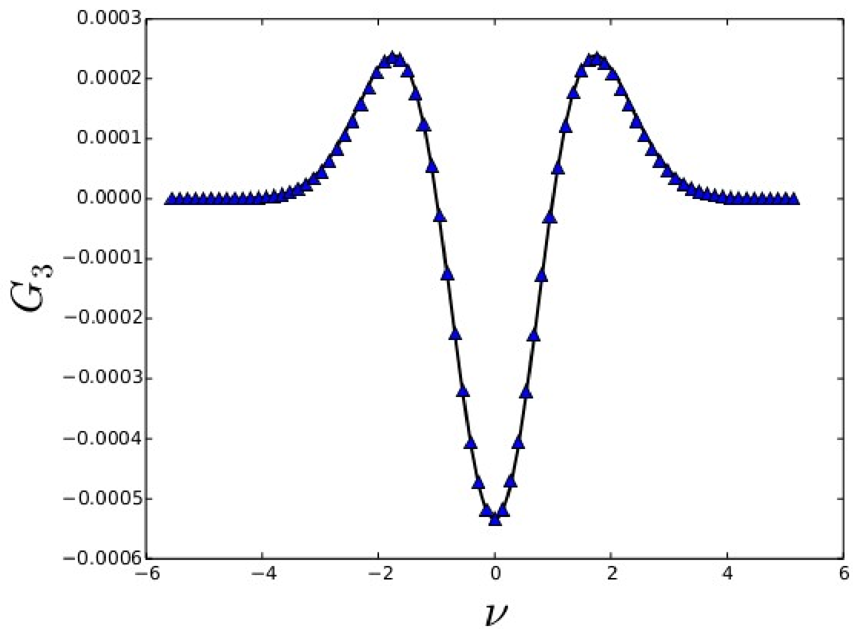


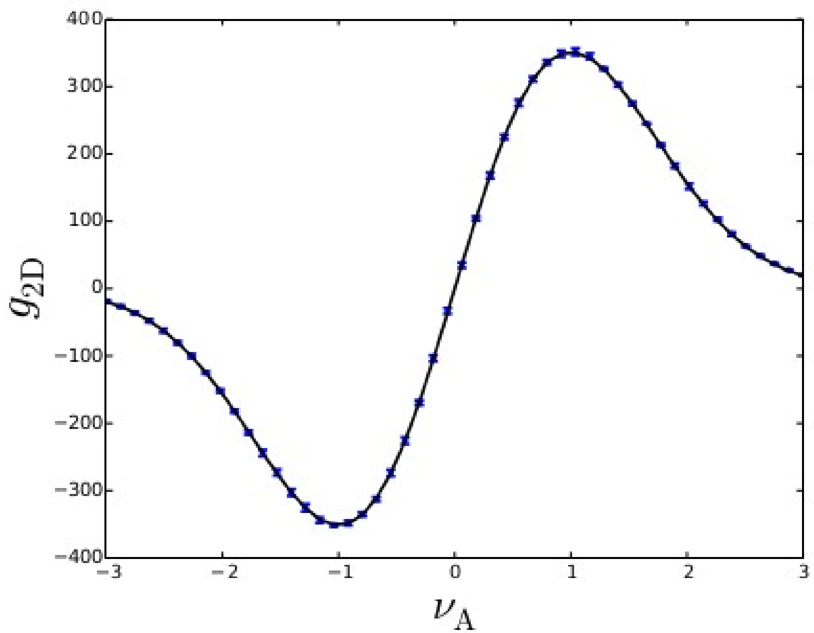
Example – Sphericity of knots $\Psi = \dfrac{\pi^{1/3}(6V)^{2/3}}{A}$



Genus - Information Content

- For a Gaussian field the genus curve shape is fixed, only the amplitude carries information





$$G_3(\nu) = \frac{\lambda^3}{\sqrt{2\pi}}(u^2 - 1)\exp\left[-\frac{1}{2}u^2\right]$$

$$u = \nu/\sqrt{\sigma} \qquad \zeta(0) = \langle \nu^2 \rangle$$

$$\lambda = \sqrt{|\zeta''(0)|/[2\pi\zeta(0)]} \qquad |\zeta''(0)| = \langle \nu_{;i}^2 \rangle$$

$$g_{2D} = \frac{1}{2(2\pi)^{3/2}}\frac{\sigma_1^2}{\sigma_0^2}\nu\exp[-\nu^2/2]$$

$$\sigma_0^2 = \sum_{\ell=1}^{\infty}(2\ell+1)\mathcal{C}_\ell$$

$$\sigma_1^2 = \sum_{\ell=1}^{\infty}(2\ell+1)\ell(\ell+1)\mathcal{C}_\ell$$

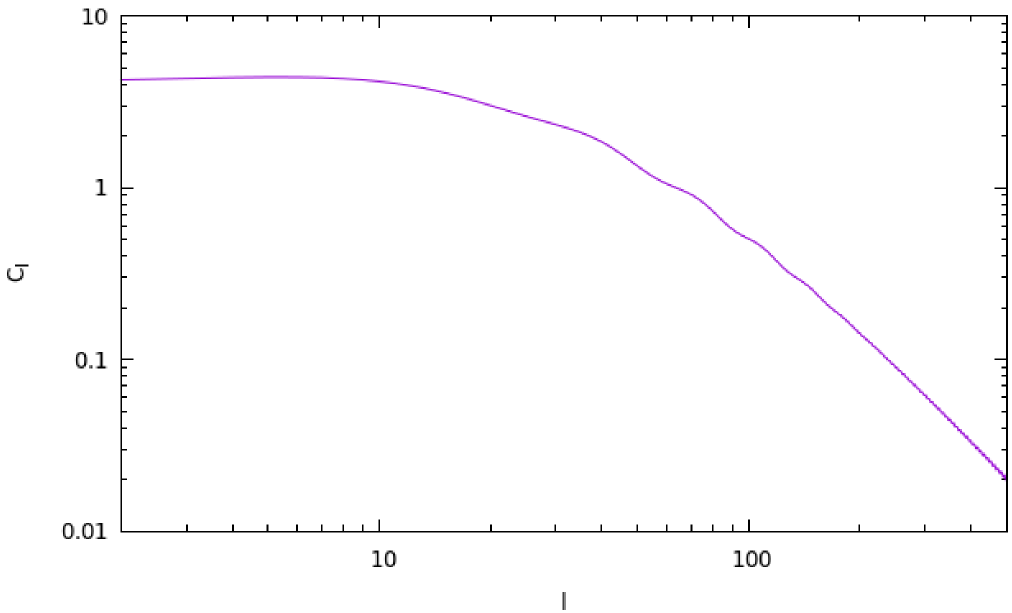
Genus – Information Content

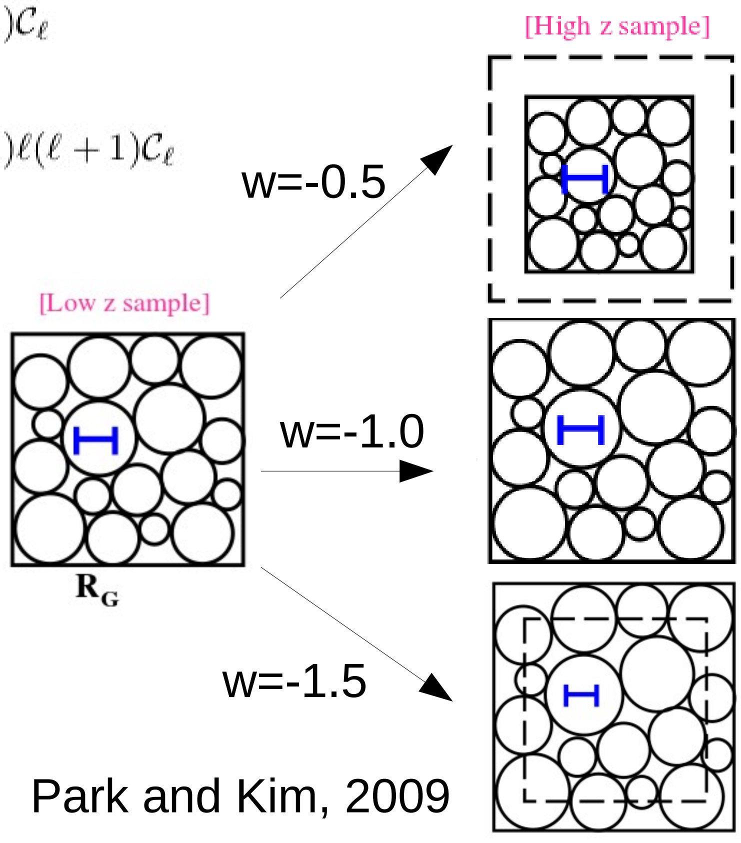
- The genus amplitude carries cosmological information.
- When we smooth the density field over large scales, the genus amplitude is a conserved quantity. We can use this information for cosmological parameter estimation.

$$g_{2D} = \frac{1}{2(2\pi)^{3/2}} \frac{\sigma_1^2}{\sigma_0^2} \nu \exp[-\nu^2/2]$$

$$\sigma_0^2 = \sum_{\ell=1}^{\infty} (2\ell+1)\mathcal{C}_\ell$$

$$\sigma_1^2 = \sum_{\ell=1}^{\infty} (2\ell+1)\ell(\ell+1)\mathcal{C}_\ell$$





Park and Kim, 2009

Genus – Information Content

- What smoothing scale will optimize the signal?

$$G_3(\nu) = \frac{\lambda^3}{\sqrt{2\pi}}(u^2 - 1)\exp\left[-\frac{1}{2}u^2\right]$$

$$g_{2D} = \frac{1}{2(2\pi)^{3/2}}\frac{\sigma_1^2}{\sigma_0^2}\nu\exp[-\nu^2/2]$$

$$u = \nu/\sqrt{\sigma} \qquad \zeta(0) = \langle \nu^2 \rangle$$

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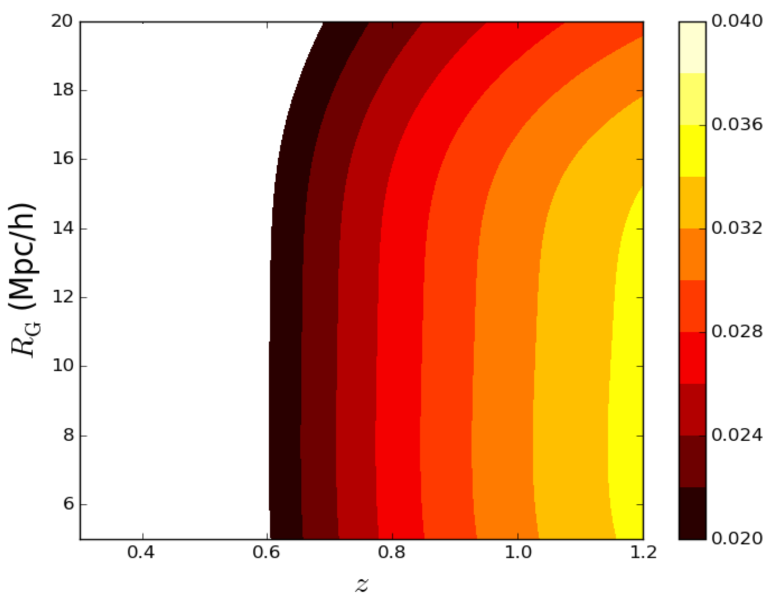
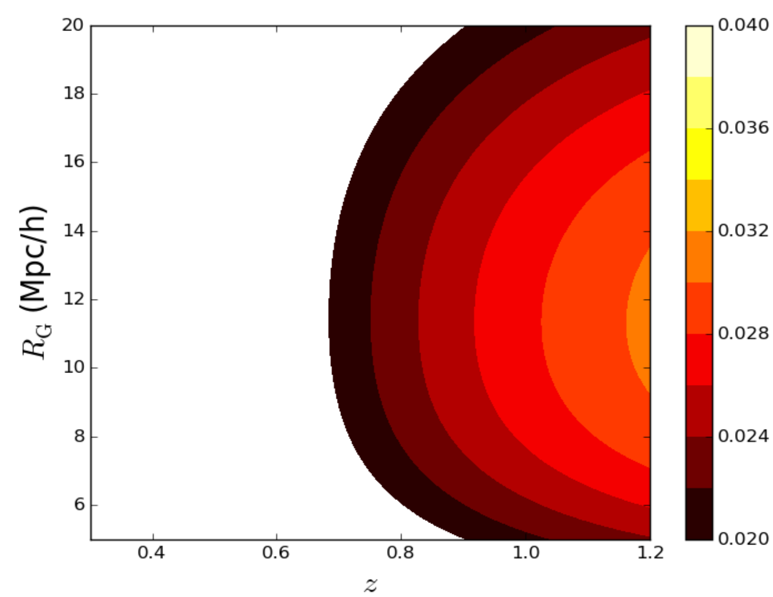
$$\boxed{\frac{A(z,\Omega_\mathrm{m},w_\mathrm{de}) - A(z=0.25,\Omega_\mathrm{m},w_\mathrm{de})}{A(z=0.25,\Omega_\mathrm{m},w_\mathrm{de})}}$$

3D



w = -0.5

2D



Genus – Information Content

- What smoothing scale will optimize the signal?

$$G_3(\nu) = \frac{\lambda^3}{\sqrt{2\pi}}(u^2-1)\exp\left[-\frac{1}{2}u^2\right]$$

$$g_{2\mathrm{D}} = \frac{1}{2(2\pi)^{3/2}}\frac{\sigma_1^2}{\sigma_0^2}\nu\exp[-\nu^2/2]$$

$$u = \nu/\sqrt{\sigma} \qquad \zeta(0) = \langle \nu^2 \rangle$$

$$\lambda = \sqrt{|\zeta''(0)|/[2\pi\zeta(0)]} \qquad |\zeta''(0)| = \langle \nu_{,i}^2 \rangle$$

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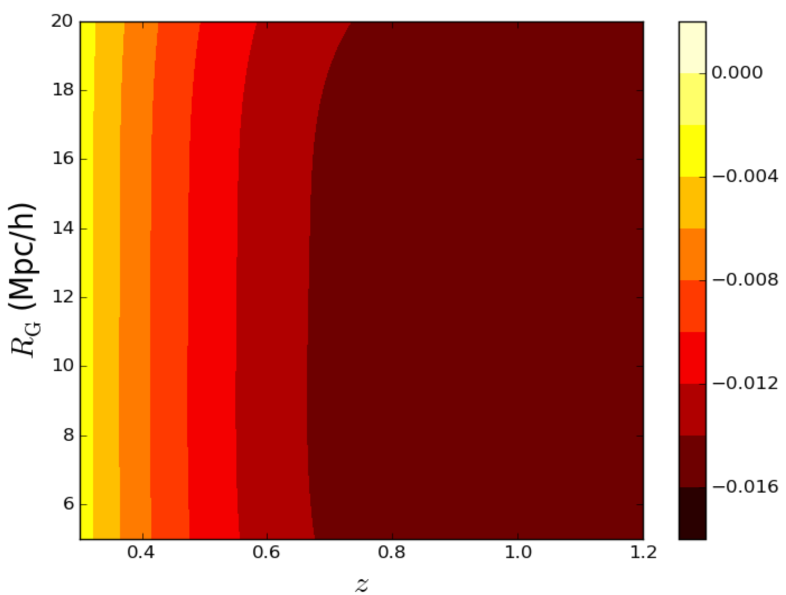
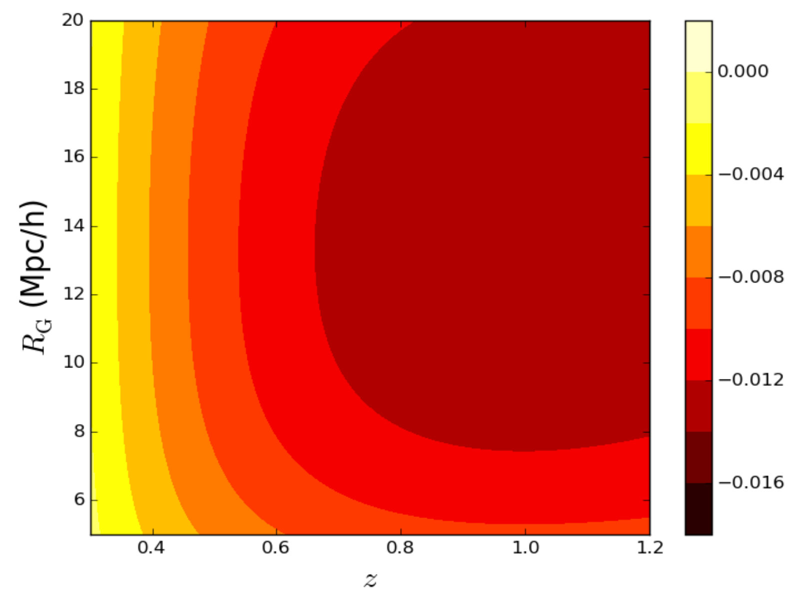
3D



2D

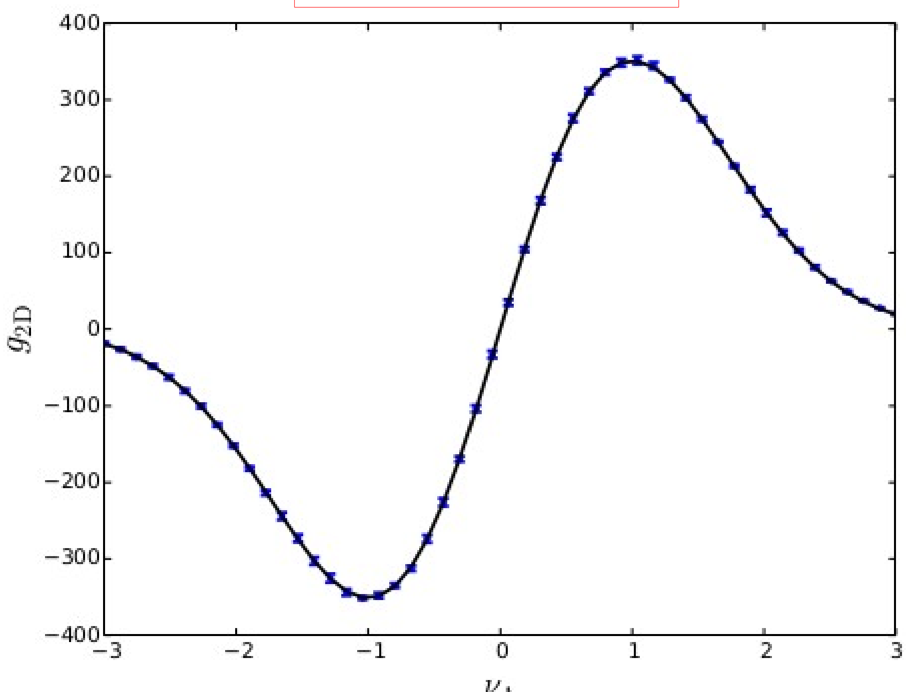


w = -1.5

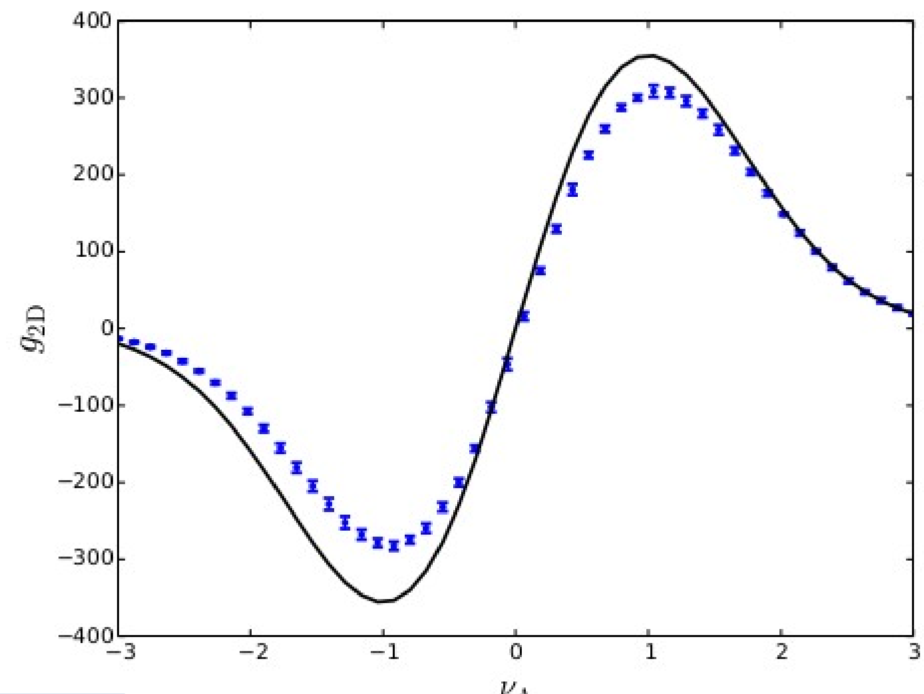
Genus – Information content

- For a non-Gaussian field there is further information in the shape of the genus curve

Gaussian field



HR4 – particle data



$$g_{2D}(\nu_A) = A e^{-\nu^2/2}\left[H_1(\nu_A) + \left[\frac{2}{3}\left(S^{(1)} - S^{(0)}\right) H_2(\nu_A) + \right.\right.$$
$$\left.\left. + \frac{1}{3}\left(S^{(2)} - S^{(0)}\right)\right] \sigma_0 + \mathcal{O}(\sigma_0^2)\right]$$

$$S^{(0)} = \frac{\langle \delta^3 \rangle}{\sigma_0^4}$$

$$S^{(1)} = -\frac{3}{4}\frac{\langle \delta^2 (\nabla^2 \delta)\rangle}{\sigma_0^2 \sigma_1^2}$$

$$S^{(2)} = -3\frac{\langle (\nabla\delta.\nabla\delta)(\nabla^2\delta)\rangle}{\sigma_1^4}$$

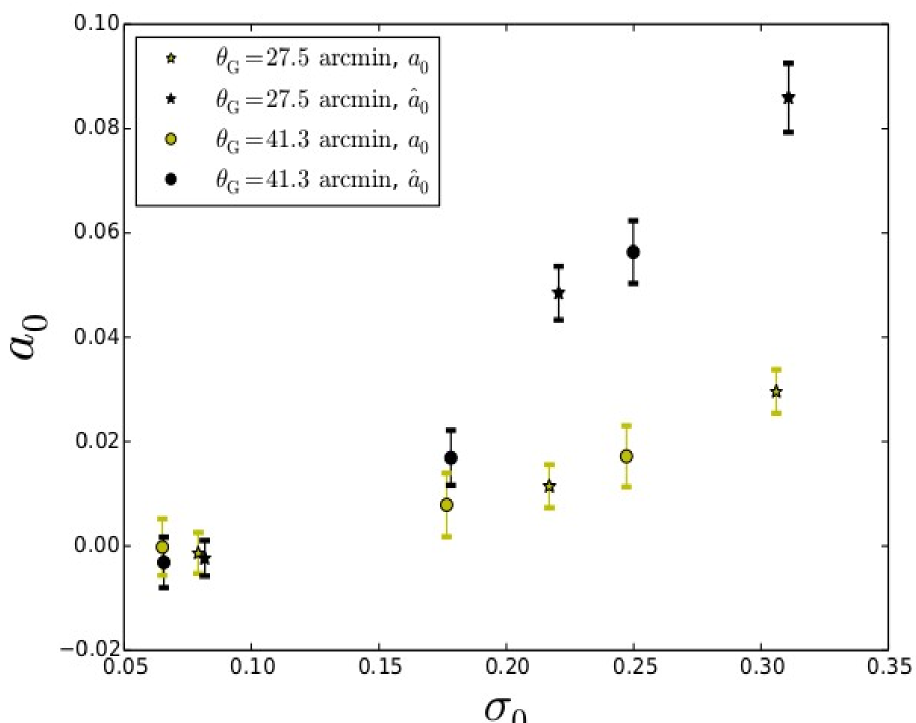
Genus – Information content

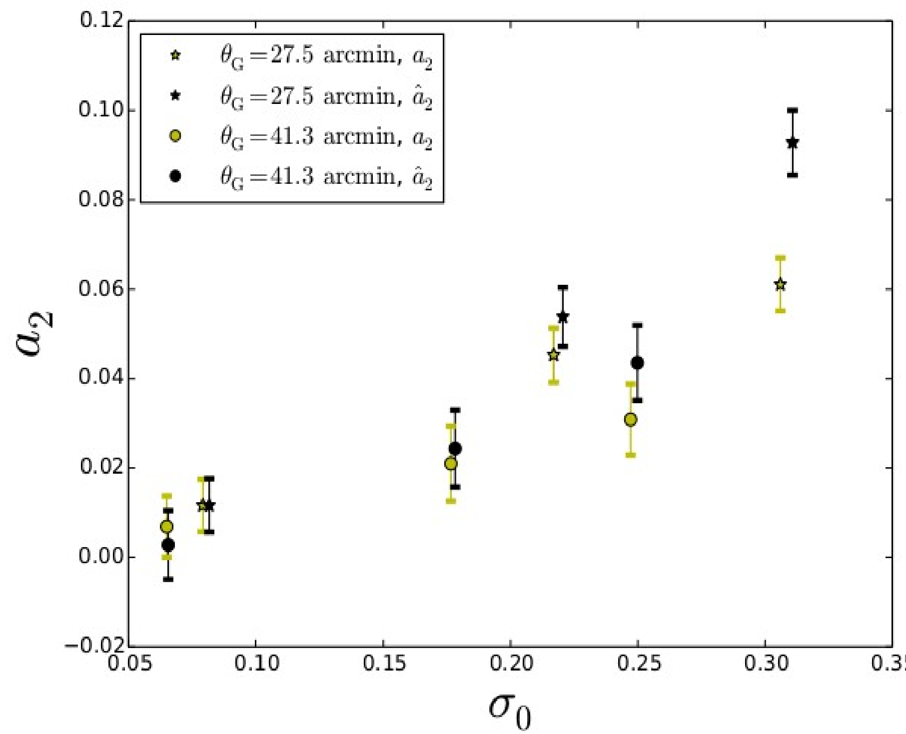
- For a non-Gaussian field there is further information in the shape of the genus curve

$$g_{2D}(\nu_A) = Ae^{-\nu^2/2}\left[H_1(\nu_A) + \left[\frac{2}{3}\left(S^{(1)} - S^{(0)}\right)H_2(\nu_A) + \right.\right.$$
$$\left.\left. + \frac{1}{3}\left(S^{(2)} - S^{(0)}\right)\right]\sigma_0 + \mathcal{O}(\sigma_0^2)\right]$$

$$S^{(0)} = \frac{\langle\delta^3\rangle}{\sigma_0^4} \qquad S^{(1)} = -\frac{3}{4}\frac{\langle\delta^2(\nabla^2\delta)\rangle}{\sigma_0^2\sigma_1^2}$$
$$S^{(2)} = -3\frac{\langle(\nabla\delta.\nabla\delta)(\nabla^2\delta)\rangle}{\sigma_1^4}$$

$$a_0 = \frac{1}{3}\left(S^{(2)} - S^{(0)}\right)\sigma_0 \qquad a_2 = \frac{1}{3}\left(S^{(1)} - S^{(0)}\right)\sigma_0$$





Leading order – Matsubara (2003), Arbitrary order – Gay et al. (2011)

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2D Minkowski Functionals

- We wish to extract cosmological information from the amplitude and shape of the genus. Before we apply the statistic to data we must understand how it is affected by various systematics that might bias our analysis.

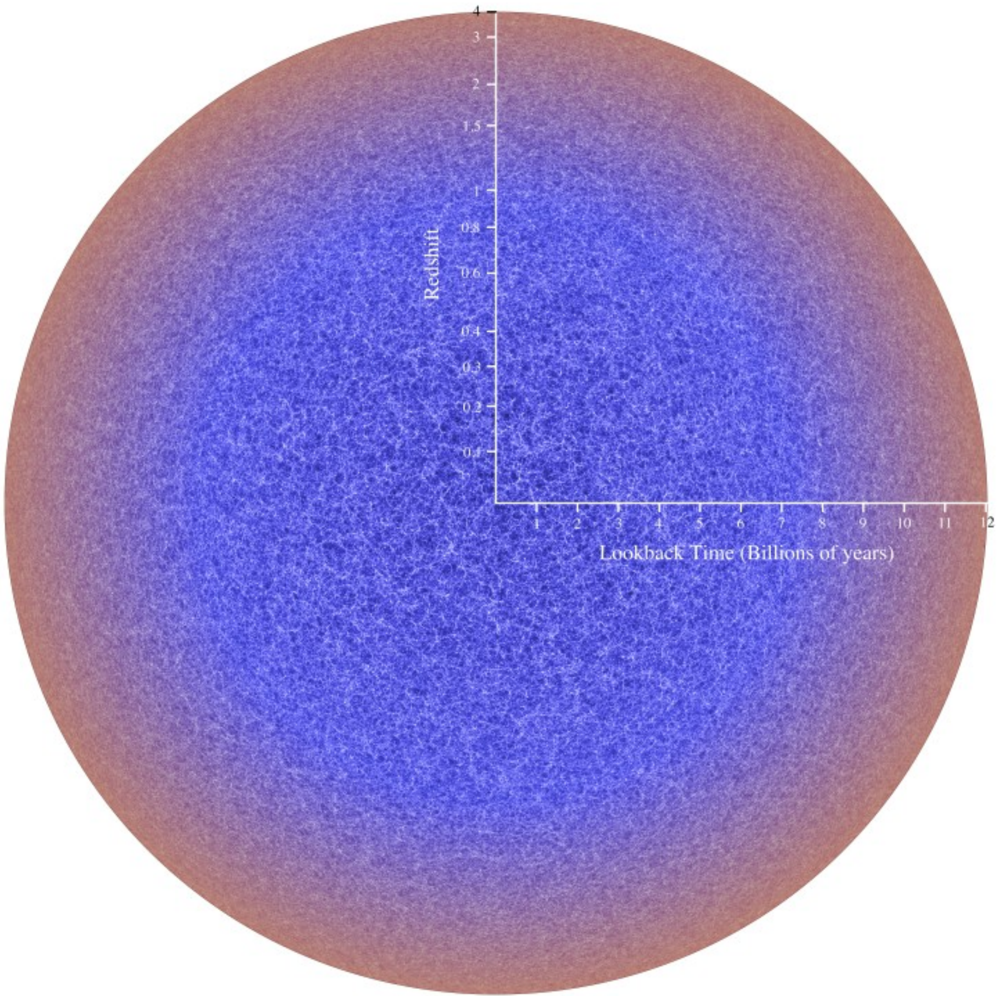
- N-body simulations are used to study how the genus is modified by the following processes

  - Redshift space distortion
  - Finite pixel size
  - Shot noise/Halo bias

$$\Delta\hat{g}_{\rm 2D}(\nu_{\rm A}, a_{0-4}) = A \exp[-\nu_{\rm A}^2/2] \sum_{i=0}^{4} a_i H_{\rm i}(\nu_{\rm A})$$

$$A = \frac{1}{2(2\pi)^{3/2}} \frac{\sigma_1^2}{\sigma_0^2}$$

$$H_0(x) = 1$$
$$H_1(x) = x$$
$$H_2(x) = x^2 - 1$$
$$H_3(x) = x^3 - 3x$$
$$H_4(x) = x^4 - 6x^2 + 3$$



Systematics – Redshift Space Distortion

- Galaxies possess a peculiar velocity along the line of sight at which we observe them. These motions cause the observed redshift to misrepresent the distance to the observer.
- On large scales, coherent in-fall produces pancake distortions, while on small scales peculiar velocities of bound objects introduce the so-call Finger of God effect.
- We study redshift space effects with the HR4 mock galaxy catalog – snapshot data
- Transform all distances to redshifts using the true cosmological redshift-distance relation, and calculate genus in redshift space by using $1 + z_{\rm obs} = (1 + z_{\rm c})\left(1 + \frac{v_{\rm los}}{c}\right)$
- In the linear limit we can estimate the amplitude of the genus in real and redshift space

$$g_{\rm 2D}^{\rm rsd}(\nu, \theta_{\rm S}) = A_{\rm RSD} g_{\rm 2D}^{\rm real}(\nu)$$

Linear result – Matsubara (1996)

Arbitrary order – Gay et al. (2013)

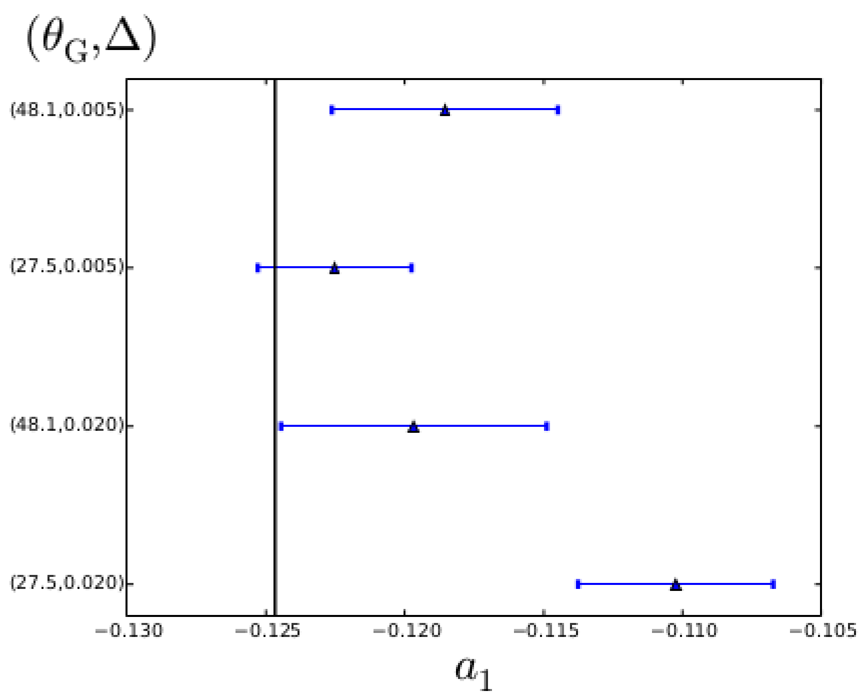
$$A_{\rm RSD} = \frac{3}{2}\sqrt{\left(1 - \frac{C_1}{C_0}\right)\left[1 - \frac{C_1}{C_0} + \left(\frac{3C_1}{C_0} - 1\right)\cos^2(\theta_{\rm S})\right]} \qquad \frac{C_1}{C_0} = \frac{1}{3}\frac{1 + 6fb^{-1}/5 + 3(fb^{-1})^2/7}{1 + 2fb^{-1}/3 + (fb^{-1})^2/5} \qquad f = \dot{D}/(HD) \simeq \Omega_{\rm mat}^{6/11}$$

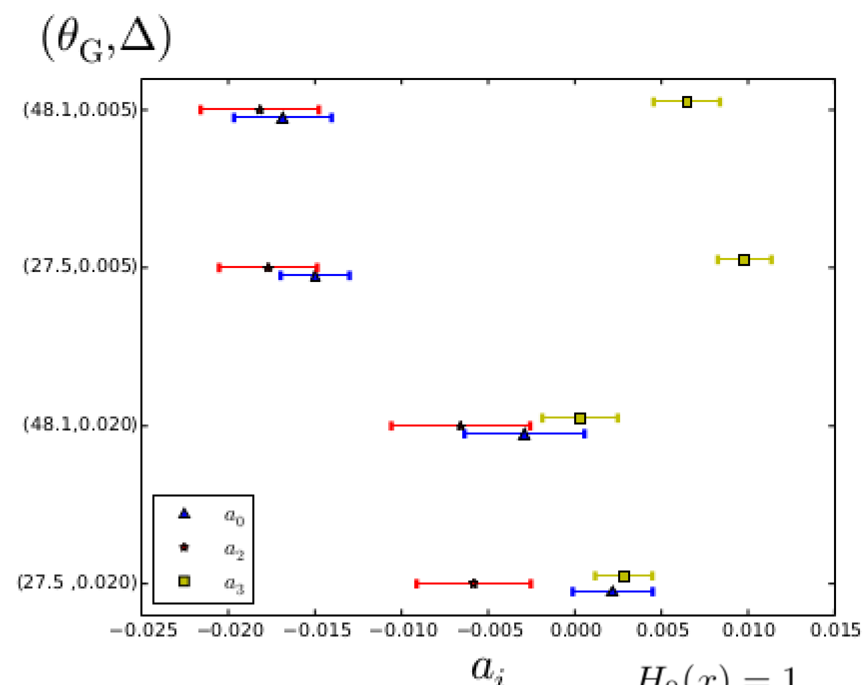
Systematics – Redshift Space Distortion

Redshift Space Distortion

$$\Delta g_{\mathrm{2D,RSD}}(\nu_{\mathrm{A}}) = g_{\mathrm{2D,RSD}}(\nu_{\mathrm{A}}) - g_{\mathrm{2D,real}}(\nu_{\mathrm{A}})$$

$$A_{\mathrm{RSD}} = \frac{3}{2}\sqrt{\left(1-\frac{C_1}{C_0}\right)\left[1-\frac{C_1}{C_0}+\left(\frac{3C_1}{C_0}-1\right)\cos^2(\theta_{\mathrm{S}})\right]} \qquad \frac{C_1}{C_0} = \frac{1}{3}\frac{1+6fb^{-1}/5+3(fb^{-1})^2/7}{1+2fb^{-1}/3+(fb^{-1})^2/5} \qquad f = \dot{D}/(HD) \simeq \Omega_{\mathrm{mat}}^{6/11}$$





$$\boxed{\Delta\hat{g}_{\mathrm{2D}}(\nu_{\mathrm{A}}, a_{0-4}) = A\exp[-\nu_{\mathrm{A}}^2/2]\sum_{i=0}^{4} a_i H_{\mathrm{i}}(\nu_{\mathrm{A}})}$$

$$A = \frac{1}{2(2\pi)^{3/2}}\frac{\sigma_1^2}{\sigma_0^2}$$

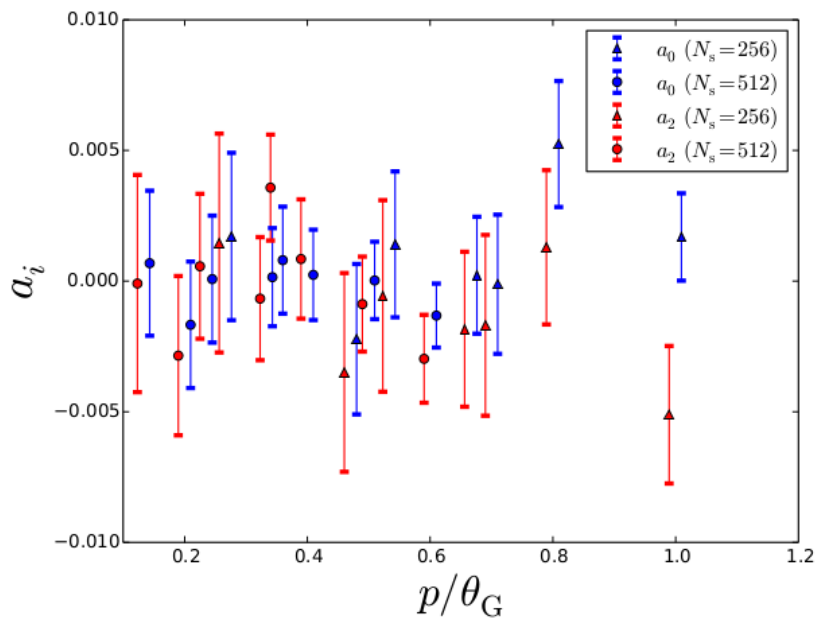
$$H_0(x) = 1$$
$$H_1(x) = x$$
$$H_2(x) = x^2 - 1$$
$$H_3(x) = x^3 - 3x$$
$$H_4(x) = x^4 - 6x^2 + 3$$

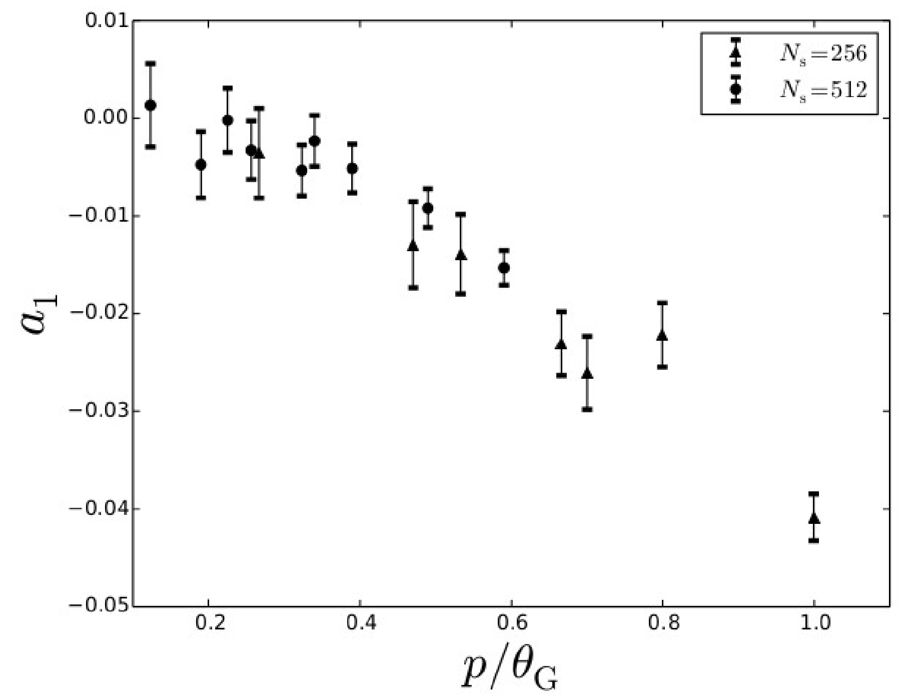
Systematics – Pixel Discretization

Finite pixel effects – numerical artifacts. A pixelated density field will perfectly represent its continuous counterpart only in the limit of vanishing pixel size. Vary the size of our pixels – study the effect on the genus curve

$$\Delta g_{\rm 2D,pix} = g_{\rm 2D}(\theta_{\rm G}, N_{\rm s}) - g_{\rm 2D}(\theta_{\rm G}, N_{\rm s} = 1024)$$

$$\Delta \hat{g}_{\rm 2D}(\nu_{\rm A}, a_{0-4}) = A \exp[-\nu_{\rm A}^2/2] \sum_{i=0}^{4} a_i H_{\rm i}(\nu_{\rm A})$$





Systematics – Shot Noise/Halo Bias

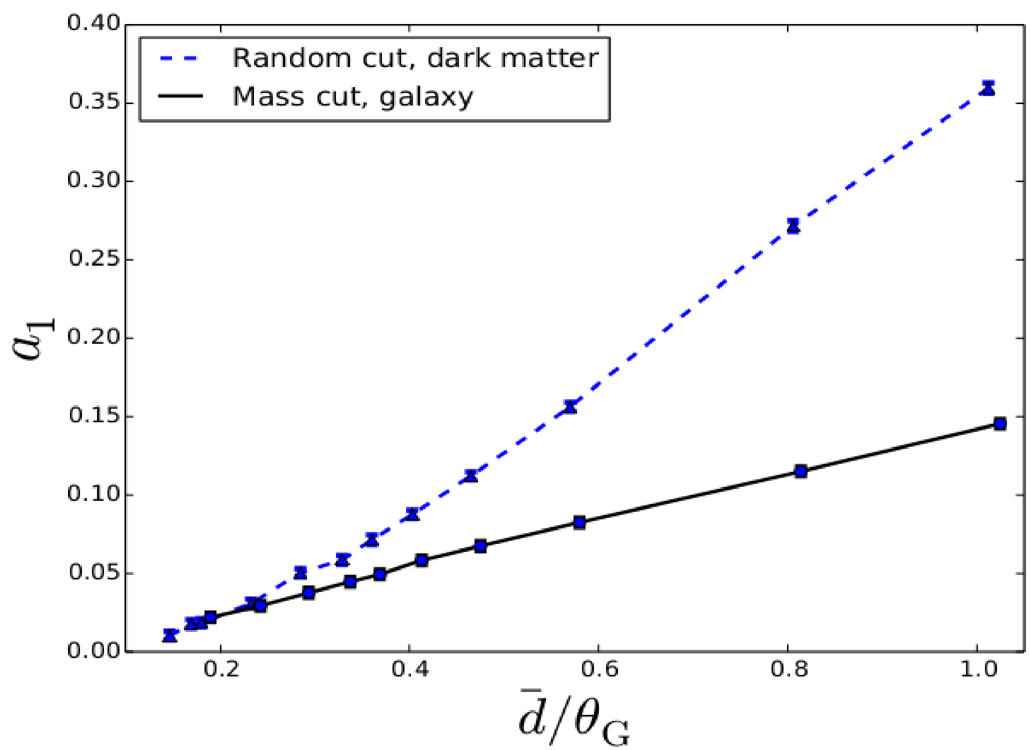
- We are using a discrete point distribution as a tracer of the underlying matter density field.
- As the number density of particles (or galaxies) decreases, the point distribution becomes a successively poorer tracer of the density.
- In addition, dark matter halos are biased tracers of the underlying matter field. High/low mass halos are positively/negatively biased with respect to the density field.
- These two effects are also entangled, as high mass halo's are also sparse and will suffer from shot noise effects
- To study the effect of shot noise on the genus, we use the HR4 simulated galaxy catalog and randomly cut galaxies
- To study halo bias, we apply mass cuts to the data.
- We compare the randomly and mass cut galaxy genus to the dark matter particle genus curve.

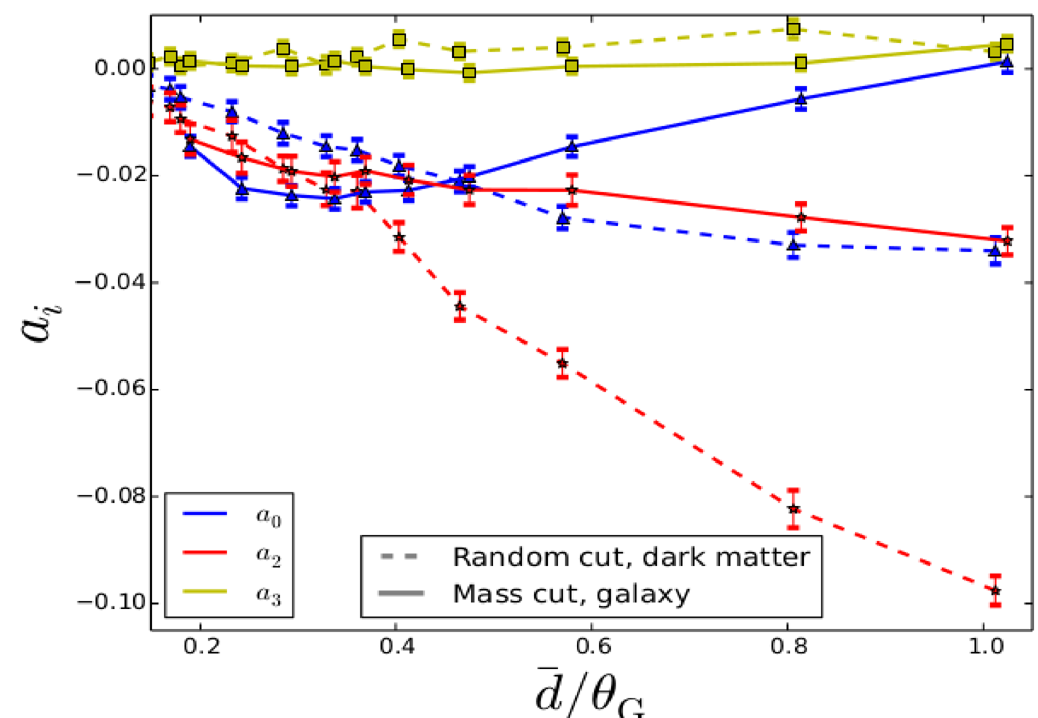
Systematics – Shot Noise/Halo Bias

- Shot noise/Halo bias

$$\Delta g_{\rm 2D,sn}(\nu_{\rm A}, \bar{d}) = g_{\rm 2D,mat}(\nu_{\rm A}, \bar{d}) - g_{\rm 2D,mat}(\nu_{\rm A}, \bar{d}_0)$$

$$\Delta g_{\rm 2D,gb}(\nu_{\rm A}, \bar{d}) = g_{\rm 2D,gal}(\nu_{\rm A}, \bar{d}) - g_{\rm 2D,mat}(\nu_{\rm A}, \bar{d}_0)$$





$$\boxed{\Delta\hat{g}_{\rm 2D}(\nu_{\rm A}, a_{0-4}) = A\exp[-\nu_{\rm A}^2/2]\sum_{i=0}^{4} a_i H_{\rm i}(\nu_{\rm A})}$$

$$\bar{d} = (10800/\pi)\sqrt{4\pi/N_{\rm p}}$$

Future Directions

- The topology of LSS contains information beyond the two point correlation function.
- Measuring the genus can provide cosmological parameter constraints by virtue of its amplitude and shape
- Extracting this information requires a thorough understanding of systematic effects. Can we control known systematics to the required precision?
- Application to mock data – additional effects to consider
  - Masks and survey boundaries
  - Photometric redshift uncertainty